

# Mathematical Simulation of Energy Coupling in Mitochondria within the Framework of the Proton–Chemical Hypothesis

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Received February 18, 1997

**Abstract** – The effect imposed by the method of transmembrane potential change on the rates of ATP synthesis and respiration in mitochondria is discussed within the framework of the proton-chemical hypothesis of coupling. Electrical and mathematical models are proposed to simulate energy coupling. A theoretical dependence of the rates of oxidative phosphorylation and mitochondrial respiration on the transmembrane potential was derived on the basis of computer analysis of the mathematical model. The theoretical dependence fits the experimental findings inconsistent with the chemiosmotic theory of energy coupling

*Key words:* mitochondria, ATP, transmembrane potential, energy coupling, model.

## INTRODUCTION

The chemiosmotic theory of energy coupling advanced by Mitchell [1] is widely recognized as the most comprehensive description of the mechanism of energy coupling in biological membranes [2]. However, the actual research has accumulated a large body of experimental findings that are either inconsistent or in clear contradiction with this theory [3-9]. This gave rise to a number of modifications of the chemiosmotic theory of energy coupling [10-12]. The chemical hypothesis of oxidative phosphorylation [4] has also been slightly modified. According to the modified chemical hypothesis, there is indeed a protonmotive force ( $\Delta\bar{\mu}_{H^+}$ ) across the coupling membrane (as suggested by Mitchell), although the protonmotive force plays only a minor function in the mechanism of energy coupling. The chemiosmotic theory of energy coupling can hardly explain the following experimental observations: (1) the rate of oxidative phosphorylation as a function of the transmembrane potential depends on the method of changing the transmembrane potential [3,4,6]; (2) the extent of

activation of mitochondrial respiration induced by a decrease in the protonmotive force may vary depending on the method of reduction of the protonmotive force (the value of the protonmotive force reduction is maintained at the same level whatever the method of reduction is used).

According to the chemiosmotic theory of energy coupling, the rates of both oxidative phosphorylation and respiration depend on the  $\Delta\bar{\mu}_{H^+}$  value but do not depend on the method of its change. These data, as well as some other results concerning the system of oxidative phosphorylation, can be consistently explained within the framework of the proton-chemical hypothesis [13-16]. This hypothesis is based on the chemical hypothesis and includes elements of the chemiosmotic theory.

The goal of this work was to construct a mathematical model designed to provide a quantitative description of the rates of ATP synthesis and respiration in mitochondria as functions of  $\Delta\bar{\mu}_{H^+}$ . The value of  $\Delta\bar{\mu}_{H^+}$  can be varied by adding uncouplers or respiration inhibitors and by changing the rate of ATP consumption.



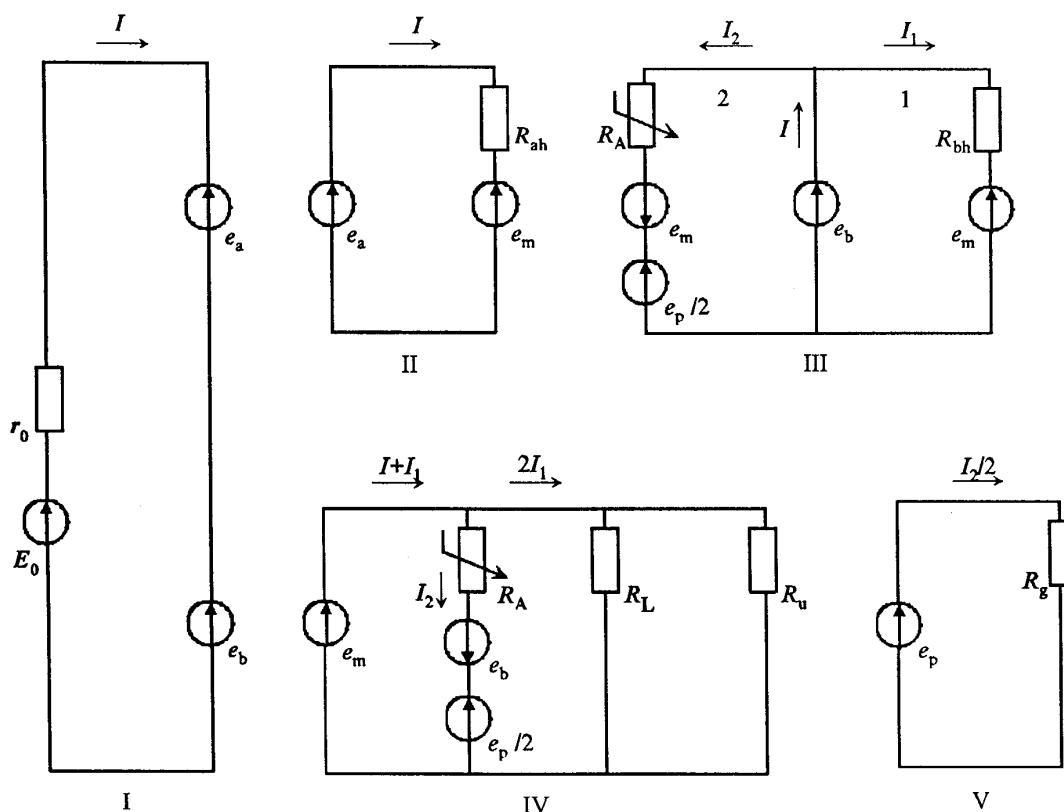


Fig. 2. Equivalent electrical circuit of two proton-chemical coupling sites (explanation in text).

between the electron donor and electron acceptor components of the coupling site [16]:

$$J_p \sim \frac{[P]^{3+}}{[P]^{5+}} \sim 10^{\frac{const - \Delta \bar{\mu}_{H^+} - \Delta E_{AB}}{2.3RT/nF}}, \quad (1)$$

where  $[P]^{3+}$  and  $[P]^{5+}$  are the phosphite and the phosphate concentrations, respectively.

The energy effect of complete reduction of phosphate to phosphite is essentially negative [18], because of the very low RP value of this redox pair [16]. This makes very unlikely the involvement of phosphite in the mechanism of energy transformation, although does not completely discard such a probability. However, it is the phosphite mechanism that allows the simplest introduction of the proton-chemical coupling [16]. Therefore, by analogy with equation (1), the exponential dependence of the rate of ATP synthesis on  $\Delta \bar{\mu}_{H^+}$  and  $\Delta E$  can be deduced from any other redox mechanism of ATP synthesis at the proton-

chemical coupling site (including one-electron or free radical mechanisms) [15]. The radical hypothesis of energy coupling has been put forward later by Schole and Schole [19]. According to this hypothesis, the role of the macroergic intermediate is played by cardiolipin, which is converted into cardiolipin-*enol*-phosphate during the redox reaction.

Most generally, the electron flow distribution between the chemiosmotic and the chemical branches of component *b* can be described within the framework of the proton-chemical principle of coupling as follows: as the coupling membrane is charged to the maximum level by the chemiosmotic mechanism, the electric current through the membrane (i.e., a kind of an electric capacitor) stops, and the RF difference  $\Delta E_{BA}$  at component *b* also attains its maximum level. This activates the electron transfer through the chemical route. Thus, the toggling is in essence thermodynamic, and may specifically involve a molecular switchover, e.g., one of the mechanisms regulating the functional activities of biological membranes by the transmembrane electric potential [17].

An electrical equivalent to the double site of proton-chemical coupling (Fig. 1, see also [16]) is shown as the five circuits in Fig. 2. The electron flow in a form of a flow of hydrogen atoms from the donor D to the acceptor B (Fig. 1) is induced by the RP difference between D and B ( $\Delta E_{BD}$ ). The RP Difference between D and B is also the driving force of the oxidative phosphorylation in general. In Fig. 2, the electric current in loop I corresponds to the electron flow through the entire coupling site, whereas the voltage source  $E_0$ , with the internal resistor  $r_0$ , represents the steady-state RP difference  $\Delta E_{BD}$  between D and B (Fig. 1). At point  $a$ , there is a steady-state RP difference between D and A ( $\Delta E_{AD}$ ), which is represented in loop I as a voltage source  $e_a$ ; and at point  $b$ , there is the RP difference  $\Delta E_{BA}$ , which is represented in loop I as a voltage source  $e_b$ . The electric current  $J$  in loop I is equal to the rate of electron transfer through the coupling site. This value is determined by the variables  $E_0$ ,  $e_a$  and  $e_b$ , as

$$J = \frac{E_0 - e_a - e_b}{r_0}. \quad (2)$$

It is obvious from Fig. 1 that the transfer of an electron from the donor D to the acceptor A in point  $a$  is accompanied by the transmembrane transfer of one  $H^+$  and generation of  $\Delta\bar{\mu}_{H^+}$ . In the electrical circuit shown in Fig. 2,  $\Delta\bar{\mu}_{H^+}$  is represented by a voltage source  $e_m$ , of loop II. In addition to  $e_m$ , loop II also includes the resistance  $R_{ah}$ , of the  $H^+$ -pump. The electric current driven in loop II by the voltage difference ( $e_a - e_m$ ) is equal to the rate of proton transport by the  $H^+$ -pump of point  $a$  and to the electric current  $J$  passing through the entire coupling site:

$$J = \frac{e_a - e_m}{R_{ah}}. \quad (3)$$

The electric current passing through point  $b$  is determined as a sum of electric currents passing through the chemiosmotic route and a parallel chemical route of ATP synthesis (Fig. 1). It is seen in loop III of the electrical circuit in Fig. 2 that the voltage  $e_b$  drives the electric current through both the right (chemiosmotic) and the left (chemical) arms of the loop (loop III.1 with current  $J_1$ , and loop III.2 with current  $J_2$ , respectively). The sum of currents  $J_1$ , and  $J_2$  passing through point  $b$  is equal to the total electric current  $J$  passing through the entire coupling

site in loop I:

$$J = J_1 + J_2. \quad (4)$$

The electric current  $J_1$  is associated with the active proton transport through point  $b$  mediated by the  $H^+$ -pump. The amplitude of the current is determined by the voltage difference ( $e_b - e_m$ ) and the pump resistance  $R_{bh}$ . According to the method of circuit currents, we can put down:

$$J_1 = \frac{e_b - e_m}{R_{bh}}. \quad (5)$$

In accordance with the two-electron mechanism of proton-chemical coupling considered in this work, synthesis of one molecule of ATP is coupled to a simultaneous transfer of two electrons and two protons along the transmembrane gradient of electrochemical potential. Hence, the amplitude of electric current  $J_2$  in loop III.2 is determined by the sum of voltages ( $e_b + e_m$ ) and by the resistance  $R_A$ , of the ATP synthase. In addition, the rate of chemical synthesis of ATP and the amplitude of electric current  $J_2$  depend on the phosphate potential  $e_p$ . This dependence can be taken into account by adding an additional source of voltage  $e_p/2$  to loop III.2, the polarity of the additional source being opposite to the polarity of  $e_b$  and  $e_m$ . The divider 2 below  $e_p$  represents the  $1ATP/2\bar{e}$  stoichiometry of the single site of the proton-chemical coupling. In this case, the value of the phosphate potential is measured in the extramitochondrial space.

Thus, according to the method of circuit currents, the electric current in loop III.2 is

$$J_2 = \frac{e_b + e_m - e_p/2}{R_A}. \quad (6)$$

On the one hand, the amplitude of current  $J_2$  is equal to twice the rate of the chemical synthesis of ATP, and on the other hand, it is equal to the rate of the proton influx to the matrix coupled to ATP synthesis.

Rather trivial calculations [16] show that in the case of the two-electron mechanism of proton-chemical coupling, the rate of ATP synthesis declines exponentially as the values of  $\Delta\bar{\mu}_{H^+}$  and/or  $\Delta E_{BA}$  decrease (see equation (1)). In the electrical circuit shown in Fig. 2, loop III, this exponential function is represented by a nonlinear depen-

dence of the ATP synthase resistance  $R_A$ , on  $e_b$  and  $e_m$ :

$$R_A = R_{A0} \cdot 10^{\frac{e_b + e_m}{30}}, \quad (7)$$

where  $R_{A0}$  is the scaling factor.

In equivalent electrical circuits,  $\Delta\bar{\mu}_{H^+}$  is represented by a voltage source  $e_m$ . The value of  $\Delta\bar{\mu}_{H^+}$  is determined by the ratio of the rate of the proton efflux by  $H^+$  pumps and the proton conductance of the coupling membrane. Loop IV in Fig. 2 represents the fact that the rate of proton efflux is equal to the sum of the rates of the proton efflux by  $H^+$  pumps at point  $a$  (current  $J$  in loop II, Fig. 2) and point  $b$  (current  $J_1$  in loop III.1, Fig. 2). Therefore, the total rate of the proton efflux is  $J + J_1$ . In the general case, the proton conductance (loop IV, Fig. 2) is determined by: (1) spontaneous proton leakage through the membrane (resistor  $R_L$ ); (2) leakage induced by addition of uncoupling agents (resistor  $R_U$ ); and (3) proton transfer through ATP synthase and phosphorylation substrate transport system (resulting resistance is numerically equal to  $R_A$ ).

According to the two-electron mechanism of proton-chemical coupling, the resulting transmembrane flow of protons, which is coupled to ATP synthesis and transport of phosphorylation substrates, is numerically equal to the corresponding electron flow  $J_2$ . Therefore, the total electric current passing through resistors  $R_U$ , and  $R_L$ , in loop IV is  $(J + J_1 - J_2)$ ; with account of equation (3), this current is equal to  $2J_1$ . On the other hand, the electric current passing through resistors  $R_U$ , and  $R_L$ , can be expressed as a function of  $e_m$ :

$$2J_1 = e_m \frac{R_U + R_L}{R_U R_L}. \quad (8)$$

The value of the phosphate potential is represented in this circuit by a voltage source  $e_p$  and is determined by the ratio of the rates of ATP synthesis at the coupling site and ATP hydrolysis in endergonic reactions. Let us assume that all endergonic reactions proceed in the extramitochondrial space. Therefore, the value of the phosphate potential in the case considered is determined for the extramitochondrial space. Hydrolysis of ATP is simulated in the electrical circuit by the resistor  $R_g$ , in loop V. In the steady state, the rate of ATP hydrolysis is equal to the rate of ATP synthesis, i.e.,  $J_2/2$ . This rate can also be expressed as the

quotient of the phosphate potential divided by the resistance  $R_g$ :

$$J_2 / 2 = \frac{e_p}{R_g}. \quad (9)$$

This brings us to a set of eight nonidentical simultaneous equations (2)-(9), which contain eight unknown variables:  $J, J_1, J_2, e_a, e_b, e_m, e_p$ , and  $R_A$ . In a more convenient form, this set of simultaneous equations can be recast as:

$$J_2 = E_0 x_1 / [R_g (x_1 + R_{ah} + r_0) / 4 + (R_{ah} + r_0) R_A + (R_{ah} + r_0 + R_A) x_1], \quad (10)$$

$$J_1 = \frac{E_0 - J_2 (R_{ah} + r_0 + R_A)}{R_{ah} + r_0}, \quad (11)$$

$$J = J_1 + J_2, \quad (12)$$

$$e_m = 2J_1 \frac{R_U R_L}{R_U + R_L}, \quad (13)$$

$$e_a = J R_{ah} + e_m, \quad (14)$$

$$e_b = E_0 - e_a - J r_0, \quad (15)$$

$$e_p = J_2 R_g / 2, \quad (16)$$

$$R_A = R_{A0} \cdot 10^{\frac{J_2 R_A + e_p / 2}{30}}, \quad (17)$$

where  $x_1 = 4 \frac{R_U R_L}{R_U + R_L} + R_{bh}$ .

Resistors  $r_0, R_{ah}, R_{bh}, R_U, R_L, R_g$ , and  $R_{A0}$ , represent individual model parameters of the system of energy coupling as they may appear in the actual experimental settings. For example, malonate-induced inhibition of succinate-dependent respiration of mitochondria can be simulated by a change in the internal resistance  $r_0$ , of the voltage source  $E_0$ . Addition of an uncoupler to the incubation medium is simulated by varying the resistance  $R_U$ , within a certain range. Activation of ATP synthesis caused by a decrease in the phosphate potential (e.g., as a result of addition of ADP or hexokinase to the incubation medium) can be simulated by a decrease in resistance  $R_g$ .

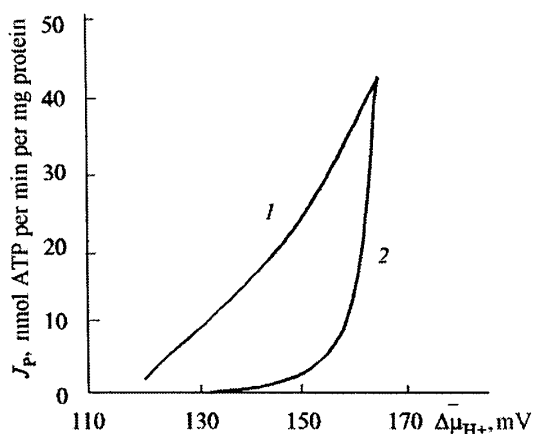


Fig. 3. Dependence of the rate of ATP synthesis on the transmembrane potential value: (1) titration with uncoupler; (2) titration with respiration inhibitor.

#### SELECTION OF PARAMETERS OF MODEL SIMULATING THE PROTON-CHEMICAL COUPLING SITE

Consider the rates of ATP synthesis and respiration at one site of proton-chemical coupling in mitochondria as functions of  $\Delta\bar{\mu}_{H^+}$ , the latter being changed by two different methods. Let the parameters of the system be defined as follows.

Let the voltage  $E_0$ , be 400 mV, i.e.,  $\sim 1/3$  of the total RP difference between the oxygen and the hydrogen electrodes.

The values of resistance are selected using the criteria of the order of magnitude of the respiratory control, the maximum extent of the uncoupler-induced activation of respiration, the low proton conductance of coupling membrane, etc. The choice of the model parameters was empirical, subject to refinement by computer simulation. The resistance values used in the model were brought to normal levels typical of one coupling site. Although the resistances are expressed in ohms, these are relative rather than absolute values.

Let the values of  $r_0$ ,  $R_{ah}$ , and  $R_{bh}$ , be equal to  $1 \Omega$  each, and the value of spontaneous proton leakage through membrane  $R_L$ , be  $100 \Omega$  (i.e., significantly higher than the  $H^+$  pump resistance). Let also the scaling factor  $R_{A0}$  of the ATP synthase resistance  $R_A$ , be  $R_{A0} = 2 \cdot 10^{11} \Omega$  (the actual value of  $R_A$ , during ATP synthesis is about  $10 \Omega$ ). The initial leakage resistance  $R_U$ , caused by addition of uncoupling agents can be taken  $10 \text{ k}\Omega$ . To the first

approximation, this value can be regarded as an infinitely large resistance  $R_U$ , in the absence of uncouplers. The value of resistance  $R_g$ , which corresponds to the mitochondrial respiration state 4 (by Chance), can be taken as large as the initial leakage resistance in the absence of uncouplers ( $R_g = 10 \text{ k}\Omega$ ). This corresponds to a negligible rate of ATP hydrolysis in the extramitochondrial space.

#### COMPUTER ANALYSIS OF A MODEL OF PROTON- CHEMICAL COUPLING

The set of simultaneous equations (10) - (17) cannot be solved explicitly, because equation (17) is selflinked and nonlinear (variable  $R_A$ , depends on its own value as an exponent index). Therefore, the method of iteration by the variable  $R_A$  was used to solve the set of simultaneous equations. The initial parameters of the equivalent electrical circuit listed above were taken as the initial parameters of the model. Calculations were performed numerically using a computer. The rates of ATP synthesis and respiration were calculated as functions of the transmembrane potential  $\Delta\bar{\mu}_{H^+}$ . The value of  $\Delta\bar{\mu}_{H^+}$  for the two dependences was changed by two different methods.

In the model analysis of the curve of the dependence of the ATP synthesis rate on the  $\Delta\bar{\mu}_{H^+}$  value, as changed experimentally by adding an uncoupler, the change of the transmembrane potential value was simulated by decreasing the  $R_U$  resistance from the initial level of  $10 \text{ k}\Omega$  to  $1 \Omega$ . The lower limit of the  $R_U$ , value ( $1 \Omega$ ) was chosen to be two orders of magnitude less the value of spontaneous proton leakage through the membrane,  $R_L$ . The value of  $R_g$ , was taken equal to  $1 \Omega$  which corresponds to  $\Delta G = 0$ .

In the model analysis of the curve of the ATP synthesis rate dependence on the  $\Delta\bar{\mu}_{H^+}$  value, as changed experimentally by the addition of a competitive inhibitor of substrate oxidation, the change in the transmembrane potential value was simulated by varying the value of the internal resistance  $r_0$ , from 1 to  $1000 \Omega$ .

The curves of the ATP synthesis rate dependence on  $\Delta\bar{\mu}_{H^+}$  for the two cases are shown in Fig. 3. It is seen that the theoretical curves decline exponentially as the transmembrane potential decreases, and are divergent (i.e., one value of  $\Delta\bar{\mu}_{H^+}$  corresponds to two different values of

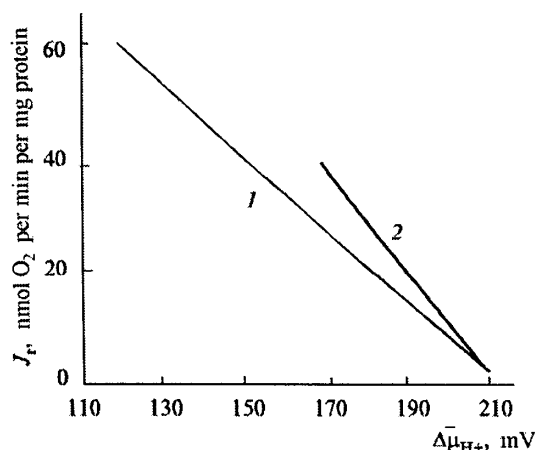


Fig. 4. Dependence of the respiration rate on the transmembrane potential value: (1) titration with uncoupler; (2) titration with hexokinase.

the ATP synthesis rate). There is a qualitative fit of the computer simulation with the experimental data obtained for mitochondria [6].

It does not follow from the chemiosmotic theory that an abrupt decrease in the ATP synthesis rate can be caused by an insignificant decrease in the transmembrane potential. Even if this were assumed to be associated with the specific kinetic characteristics of the ATP synthase, it would be impossible to explain within the framework of the chemiosmotic theory the existence of two differing values of the ATP synthesis rate at the same value of  $\Delta\bar{\mu}_{H^+}$  changed by different methods. However, this artifact is a mathematically rigorous corollary to the model of proton-chemical coupling.

Two modes of  $\Delta\bar{\mu}_{H^+}$  changing (titration with a protonophore uncoupler or titration with hexokinase) were considered during the theoretical simulation of the curve of the mitochondrial respiration rate dependence on the transmembrane potential. As noted above, titration with an uncoupler was simulated by decreasing the  $R_U$ , resistance from the initial level of  $10\text{ k}\Omega$  to  $1\ \Omega$ . Hexokinase titration was simulated by decreasing the  $R_g$ , value (resistance to ATP hydrolysis) from the initial level of  $10\text{ k}\Omega$  to  $1\ \Omega$ . Two diverging straight lines were obtained as a result of simulation (Fig. 4), which is in a qualitative agreement with certain experimental data contradicting the Mitchell's hypothesis [3, 9].

The mathematical and electrical models of proton-

chemical coupling considered in this work allow a number of thermodynamic parameters of the system of oxidative phosphorylation to be brought in correlation with each other. Computer simulation of these models provides interpretation of a large body of experimental results from the standpoint of the proton-chemical coupling. The experimental findings inconsistent with the chemiosmotic theory of energy coupling can also be explained. The possibility of simulating the physicochemical characteristics of the system of energy coupling that cannot be measured by the currently available experimental methods is an advantage of the model suggested in this work.

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